For your Eighth-Grade Summer Math Homework, you need to complete the “Test Yourself” sections of this packet (there are 17 of them). Please complete this work on graph paper--label the sections and catalog your work neatly.

There is also an answer key at the end of this packet for you to check your answers when you are done. If you score less than 80% on a “Test Yourself”, please read through the notes/explanations that are part of the assignment and make the necessary corrections.

If you are unable to complete a section and/or are confused about the processes to employ in its completion, please consult any of the following resources: your textbook, your Note-to-Self book, the digital Classroom video resources, online resources like Khan Academy, friends/family/teachers.

These are not only good practice for you as you begin 8th grade (think about admissions and placement exams, and begin to prepare to enter a world of test taking and letter grades), they are also very informative for us as we determine what skills we do need to review before you leave Arbor. Please do not save this work until the last moment and please bring this with you to the first day of school.

Mathematics

chapter 17

OVERVIEW

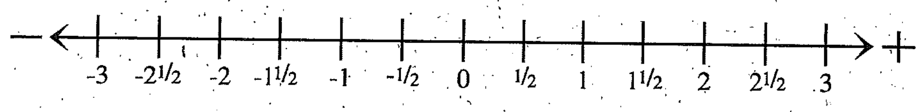
- The number line
- Decimals
- Fractions
- Percentages
- Algebra
- Equations
- Geometry
- Coordinate geometry
- Word problems
- Summing it up

Whether you love math or hate it, it's always a part of your life. Mathematics questions are found on all scholastic aptitude and achievement tests, including Catholic high school entrance exams. On the COOP exam, these questions are called Mathematics. On the HSPT, math questions include the categories of Concepts, Problem-Solving, and Quantitative Skills. On the TACHS, the questions are called Math.

In the pages that follow, we have tried to condense eight years of mathematics instruction into a comprehensive review that touches on most of the topics covered on the exams. This is only a review, not a course. If you find that you're having difficulties with any mathematic topic, talk with a teacher or refer to any of your mathematics textbooks. This chapter really helps you most by letting you know what you *don't* know, so you can focus some of your test-prep time on brushing up your skills in problem areas. The explanations that accompany the mathematics exercises are very complete. These explanations will be a big help to you, because they help you understand the processes involved in finding the right answers to mathematics questions. For extra practice with math questions, do the math sections of all the practice exams that follow.

The following sections in this part outline some of the basic mathematic rules, procedures, and formulas that you've learned over the past eight years in school. You also have an opportunity to practice your skills with some exercises, and you can judge your progress by checking your work against the answer explanations that follow the exercises. Work through these sections and the exercises carefully, and be honest with yourself about the accuracy and speed with which you solve these problems. Note which problems are difficult for you as well as those that are easy. After you've completed this section, you'll know exactly which areas you need to strengthen.

THE NUMBER LINE



A *number line* is a convenient concept to keep as a mental picture. The number line above shows whole numbers and fractions greater than zero and less than zero. Numbers increase in size as you move to the right and decrease in size as you move to the left. The number line above has an arrow at each end, meaning that the number line goes on infinitely in both positive and negative directions.

Number lines can be drawn up to aid in basic mathematical calculations. Either fractions, whole numbers, or decimals can be used to name the intervals on the line. We suggest that you use number lines when dealing with signed (+, −) numbers and inequalities.

Here is a list of a few basic rules that must be mastered for speed and accuracy in mathematical computation. You should memorize these rules:

Any number multiplied by 0 = 0.
$5 \times 0 = 0$

If 0 is divided by any number, the answer is 0.
$0 \div 2 = 0$

If 0 is added to any number, that number does not change.
$7 + 0 = 7$

If 0 is subtracted from any number, that number does not change.
$4 - 0 = 4$

If a number is multiplied by 1, that number does not change.
$3 \times 1 = 3$

If a number is divided by 1, that number does not change.
$6 \div 1 = 6$

A number added to itself is doubled.
$4 + 4 = 8$

If a number is subtracted from itself, the answer is 0.
$9 - 9 = 0$

If a number is divided by itself, the answer is 1.
$8 \div 8 = 1$

If you have memorized these rules, you should be able to write the answers to the questions in the following exercise as fast as you can read the questions.

Test Yourself 1

Answers follow Test Yourself 19.

1. $1 - 1 =$
2. $3 \div 1 =$
3. $6 \times 0 =$
4. $6 - 0 =$
5. $0 \div 8 =$
6. $9 \times 1 =$
7. $5 + 0 =$
8. $4 - 0 =$
9. $2 \div 1 =$
10. $7 - 7 =$
11. $8 \times 0 =$
12. $0 \div 4 =$
13. $1 + 0 =$
14. $3 - 0 =$
15. $5 \times 1 =$
16. $9 \div 1 =$
17. $6 + 6 =$
18. $4 - 4 =$
19. $5 \div 5 =$
20. $6 \times 1 =$

The more rules, procedures, and formulas you are able to memorize, the easier it will be to solve mathematical problems on your exam and throughout life. Become thoroughly familiar with the rules in this section, and try to commit to memory as many as possible.

When multiplying a number by 10, 100, 1000, etc., move the decimal point to the right a number of spaces equal to the number of zeros in the multiplier. If the number being multiplied is a whole number, push the decimal point to the right by inserting the appropriate number of zeros.

$$0.36 \times 100 = 36$$
$$1.2 \times 10 = 12$$
$$5. \times 10 = 50$$
$$60.423 \times 100 = 6042.3$$

When dividing a number by 10, 100, 1000, etc., again count the zeros, but this time move the decimal point to the left.

$$123. \div 100 = 1.23$$
$$352.8 \div 10 = 35.28$$
$$16. \div 100 = 0.16$$
$$7. \div 1000 = 0.007$$

Test Yourself 2

1. $18 \times 10 =$
2. $5 \div 100 =$
3. $1.3 \times 1000 =$
4. $3.62 \times 10 =$
5. $9.86 \div 10 =$
6. $0.12 \div 100 =$
7. $4.5 \times 10 =$
8. $83.28 \div 1000 =$
9. $761 \times 100 =$
10. $68.86 \div 10 =$

DECIMALS

Remember decimals are a way of writing fractions using tenths, hundredths, thousandths, etc.

Decimals are a way of writing fractions using tenths, hundredths, thousandths, and so forth. If you can count money, make change, or understand a batting average, decimals should present no problem.

When writing decimals, the most important step is placing the decimal point. The whole system is based on its location. Remember the decimal places?

1,	2	3	6,	5	4	0	.	1	3	2	4	5	6
MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES	DECIMAL POINT	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS	HUNDRED THOUSANDTHS	MILLIONTHS

When adding or subtracting decimals, it is most important to keep the decimal points in line. After the decimal points are aligned, proceed with the problem in exactly the same way as with whole numbers, simply maintaining the location of the decimal point.

Example: Add 36.08 + 745 + 4.362 + 58.6 + 0.0061.

Solution:

$$\begin{array}{r@{.}l} 36 & 08 \\ 745 & \\ 4 & 362 \\ 58 & 6 \\ +\quad 0 & 0061 \\ \hline 844 & 0481 \end{array}$$

If you find it easier, you may fill in the spaces with zeroes. The answer will be unchanged.

$$\begin{array}{r} 036.0800 \\ 745.0000 \\ 004.3620 \\ 058.6000 \\ +\quad 000.0061 \\ \hline 844.0481 \end{array}$$

Example: Subtract 7.928 from 82.1.

Solution:

$$\begin{array}{r} 82.1 \\ -\ \ 7.928 \\ \hline 74.172 \end{array} \qquad \begin{array}{r} 82.100 \\ -\ \ 7.928 \\ \hline 74.172 \end{array}$$

Test Yourself 3

1. 1.52 + 0.389 + 42.9 =
2. 0.6831 + 0.01 + 4.26 + 98 =
3. 84 − 1.9 =
4. 3.25 + 5.66 + 9.1 =
5. 17 − 12.81 =
6. 46.33 − 12.1 =
7. 51 + 7.86 + 42.003 =
8. 35.4 − 18.21 =
9. 0.85 − 0.16 =
10. 7.6 + 0.32 + 830 =

When multiplying decimals, you can ignore the decimal points until you reach the product. Then the placement of the decimal point is dependent on the sum of the places to the right of the decimal point in both the multiplier and number being multiplied.

$$\begin{array}{rl} 1.482 & \text{(3 places to the right of decimal point)} \\ \times\ 0.16 & \text{(2 places to the right of decimal point)} \\ \hline 8892 & \\ 14820 & \\ \hline 0.23712 & \text{(5 places to the right of decimal point)} \end{array}$$

You cannot divide by a decimal. If the divisor is a decimal, you must move the decimal point to the right until the divisor becomes a whole number, an integer. Count the number of spaces by which you moved the decimal point to the right and move the decimal point in the dividend (the number being divided) the same number of spaces to the right. The decimal point in the answer should be directly above the decimal point in the dividend.

$$\begin{array}{r} 70.2 \\ 0.06\overline{)4.212} \end{array}$$

Decimal point moves two spaces to the right.

Test Yourself 4

Solve the following problems.

1. $3.62 \times 5.6 =$
2. $92 \times 0.11 =$
3. $18 \div 0.3 =$
4. $1.5 \times 0.9 =$
5. $7.55 \div 5 =$
6. $6.42 \div 2.14 =$
7. $12.01 \times 3 =$
8. $24.82 \div 7.3 =$
9. $0.486 \div 0.2 =$
10. $0.21 \times 12 =$

FRACTIONS

Fractions are used when we wish to indicate parts of things. A fraction consists of a numerator and a denominator.

$$\frac{3}{4} \begin{array}{c} \leftarrow \text{numerator} \rightarrow \\ \leftarrow \text{denominator} \rightarrow \end{array} \frac{7}{8}$$

The denominator tells you how many equal parts the object or number has been divided into, and the numerator tells how many of those parts we are concerned with.

Example: Divide a baseball game, a football game, and a hockey game into convenient numbers of parts. Write a fraction to answer each equation.

1. If a pitcher played two innings, how much of the whole baseball game did he play?
2. If a quarterback played three parts of a football game, how much of the whole game did he play?
3. If a goalie played two parts of a hockey game, how much of the whole game did he play?

Solution 1: A baseball game is conveniently divided into nine parts (each an inning). The pitcher pitched two innings. Therefore, he played $\frac{2}{9}$ of the game. The denominator represents the nine parts the game is divided into; the numerator, the two parts we are concerned with.

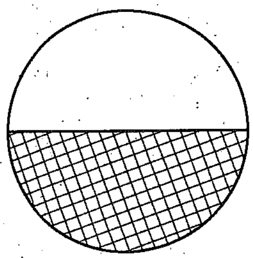
Solution 2: Similarly, there are four quarters in a football game, and a quarterback playing three of those quarters plays in $\frac{3}{4}$ of the game.

Solution 3: There are three periods in hockey, and the goalie played in two of them. Therefore, he played in $\frac{2}{3}$ of the game.

Equivalent Fractions

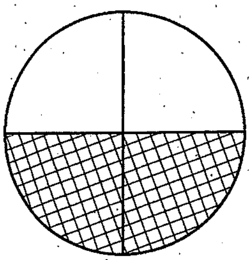
Fractions having different denominators and numerators might actually represent the same amount. Such fractions are equivalent fractions.

For example, the following circle is divided into two equal parts. Write a fraction to indicate how much of the circle is shaded.



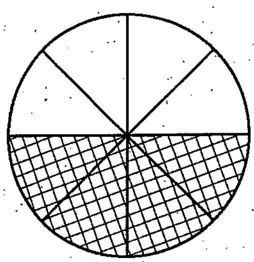
$\frac{\text{1 shaded}}{\text{2 parts}} = \frac{1}{2}$ of the circle is shaded.

The circle below is divided into four equal parts. Write a fraction to indicate how much of the circle is shaded.



$\frac{\text{2 shaded}}{\text{4 parts}} = \frac{2}{4}$ of the circle are shaded.

This circle is divided into eight equal parts. Write a fraction to indicate how much of the circle is shaded.



$\frac{\text{4 shaded}}{\text{8 parts}} = \frac{4}{8}$ of the circle are shaded.

In each circle, the same amount was shaded. This should show you that there is more than one way to indicate one half of something.

The fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ that you wrote are *equivalent fractions* because they all represent the same amount. Notice that the denominator is twice as large as the numerator in every case. Any fraction you write that has a denominator that is exactly twice as large as the numerator will be equivalent to $\frac{1}{2}$.

Example: Write other fractions equivalent to $\frac{1}{2}$.

Solution: Any fraction that has a denominator that is twice as large as the numerator: $\frac{3}{6}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{32}{64}$, etc.

Example: Write other fractions equivalent to $\frac{1}{4}$.

Solution: Any fraction that has a denominator that is four times as large as the numerator: $\frac{2}{8}$, $\frac{4}{16}$, $\frac{5}{20}$, $\frac{15}{60}$, etc.

Example: Write other fractions equivalent to $\frac{2}{3}$.

Solution: Any fraction that has a denominator that is one and one-half times as large as the numerator: $\frac{4}{6}$, $\frac{10}{15}$, $\frac{14}{21}$, $\frac{16}{24}$, etc.

When the numerator and denominator of a fraction cannot be divided evenly by the same whole number (other than 1), the fraction is said to be in simplest forms. In the examples above, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{2}{3}$ are in simplest form.

To write equivalent fractions where the numerator is not 1 requires one more step.

Example: What is the equivalent fraction for $\frac{4}{5}$ using 10 as a denominator?

Solution: Each $\frac{1}{5}$ is equivalent to $\frac{2}{10}$; therefore, $\frac{4}{5}$ is equivalent to $\frac{8}{10}$.

The quickest way to find an equivalent fraction is to divide the denominator of the fraction you want by the denominator you know. Take the result and multiply it by the numerator of the fraction you know. This becomes the numerator of the equivalent fraction.

Example: Rename $\frac{3}{8}$ as an equivalent fraction having 16 as a denominator.

Solution: $16 \div 8 = 2$; $2 \times 3 = 6$ *Answer:* $\frac{6}{16}$

Example: Rename $\frac{3}{4}$ as equivalent fractions having 8, 12, 24, and 32 as denominators.

Solution: $\frac{3}{4} = \frac{6}{8}$ $(8 \div 4 = 2; 2 \times 3 = 6)$

$\frac{3}{4} = \frac{9}{12}$ $(12 \div 4 = 3; 3 \times 3 = 9)$

$\frac{3}{4} = \frac{18}{24}$ $(24 \div 4 = 6; 6 \times 3 = 18)$

$\frac{3}{4} = \frac{24}{32}$ $(32 \div 4 = 8; 8 \times 3 = 24)$

A fraction that has a numerator greater than the denominator is called an *improper fraction*. A number expressed as an integer together with a proper fraction is called a *mixed number*.

Examples of improper fractions include $\frac{3}{2}$, $\frac{12}{7}$, and $\frac{9}{5}$. Note that each is in simplest form because the numerator and denominator cannot be divided evenly by a number other than 1.

Examples of mixed numbers include $1\frac{1}{2}$, $1\frac{5}{7}$, and $1\frac{4}{5}$. These are called mixed numbers because they have a whole number part and a fractional part. These mixed numbers are equivalent to the improper fractions given previously. To rename a mixed number as an improper fraction is easy.

Example: Rename $2\frac{1}{4}$ as an improper fraction.

Solution: The whole number 2 contains 8 fourths. Add to it the $\frac{1}{4}$ to write the equivalent fraction $\frac{9}{4}$.

An alternative way of figuring this is to multiply the denominator of the fraction by the whole number and add the numerator.

Example: Rename $2\frac{1}{4}$ as an improper fraction.

Solution: $4 \times 2 = 8 + 1 = 9; \frac{9}{4}$

To rename an improper fraction as a mixed number, just proceed backward.

Example: Rename $\frac{9}{4}$ as a mixed number.

Solution: Divide the numerator by the denominator and use the remainder (R) as the fraction:

$$9 \div 4 = 2 \text{ R1 or } 9 \div 4 = 2\frac{1}{4}$$

Adding and Subtracting Fractions

To add fractions having the same denominators, simply add the numerators and keep the common denominator.

Example: Add $\frac{1}{4} + \frac{3}{4} + \frac{3}{4}$.

Solution: The denominators are the same, so just add the numerators to arrive at the answer, $\frac{7}{4}$ or $1\frac{3}{4}$.

To find the difference between two fractions having the same denominators, simply subtract the numerators, leaving the denominators alone.

Example: Find the difference between $\frac{7}{8}$ and $\frac{3}{8}$.

Solution: $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$. Simplified to simplest form $\frac{4}{8} = \frac{1}{2}$.

To add or subtract fractions having different denominators, you will have to find a *common denominator*. A common denominator is a number that can be divided by the denominators of all the fractions in the problem without a remainder.

Example: Find a common denominator for $\frac{1}{4}$ and $\frac{1}{3}$.

Solution: 12 can be divided by both 4 and 3:

$$\frac{1}{4} \text{ is equivalent to } \frac{3}{12}$$
$$\frac{1}{3} \text{ is equivalent to } \frac{4}{12}$$

We can now add the fractions because we have written equivalent fractions with a common denominator.

$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

Therefore:

$$\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

Seven twelfths is in simplest form because 7 and 12 do not have a whole number (other than 1) by which they are both divisible.

Example: Add $\frac{3}{8}, \frac{5}{6}, \frac{1}{4}$, and $\frac{2}{3}$.

Solution: Find a number into which all denominators will divide evenly. For 8, 6, 4, and 3, the best choice is 24. Now convert each fraction to an equivalent fraction having a denominator of 24:

$$\frac{3}{8} = \frac{9}{24} \ (24 \div 8 = 3; 3 \times 3 = 9)$$

$$\frac{5}{6} = \frac{20}{24} \ (24 \div 6 = 4; 4 \times 5 = 20)$$

$$\frac{1}{4} = \frac{6}{24} \ (24 \div 4 = 6; 6 \times 1 = 6)$$

$$\frac{2}{3} = \frac{16}{24} \ (24 \div 3 = 8; 8 \times 2 = 16)$$

Now add the fractions:

$$\frac{9}{24} + \frac{20}{24} + \frac{6}{24} + \frac{16}{24} = \frac{51}{24}$$

The answer, $\frac{51}{24}$, is an improper fraction; that is, the numerator is greater than the denominator. To rename the answer to a mixed number, divide the numerator by the denominator and express the remainder as a fraction.

$$\frac{51}{24} = 51 \div 24 = 2\frac{3}{24} = 2\frac{1}{8}$$

Test Yourself 5

Express your answers as simple mixed numbers.

1. $\frac{2}{4}+\frac{3}{5}+\frac{1}{2}=$
2. $\frac{6}{8}-\frac{2}{4}=$
3. $\frac{1}{3}+\frac{1}{2}=$
4. $\frac{4}{5}-\frac{3}{5}=$
5. $\frac{7}{8}+\frac{3}{4}+\frac{1}{3}=$
6. $\frac{1}{2}+\frac{1}{4}+\frac{2}{3}=$
7. $\frac{5}{6}-\frac{1}{2}=$
8. $\frac{5}{8}-\frac{1}{3}=$
9. $\frac{5}{12}+\frac{3}{4}=$
10. $\frac{8}{9}-\frac{2}{3}=$

Multiplying and Dividing Fractions

When multiplying fractions, multiply numerators by numerators and denominators by denominators.

$$\frac{3}{5}\times\frac{4}{7}\times\frac{1}{5}=\frac{3\times4\times1}{5\times7\times5}=\frac{12}{175}$$

When multiplying fractions, try to work with numbers that are as small as possible. You can make numbers smaller by dividing out common factors. Do this by dividing the numerator of any one fraction and the denominator of any one fraction by the same number.

$$\frac{{}^{1}\cancel{3}}{\cancel{4}_{2}}\times\frac{{}^{1}\cancel{2}}{\cancel{9}_{3}}=\frac{1\times1}{2\times3}=\frac{1}{6}$$

In this case, the numerator of the first fraction and the denominator of the other fraction were divided by 3, while the denominator of the first fraction and the numerator of the other fraction were divided by 2.

To divide by a fraction, multiply by the reciprocal of the divisor.

$$\frac{3}{16}\div\frac{1}{8}=\frac{3}{\cancel{16}_{2}}\times\frac{\cancel{8}^{1}}{1}=\frac{3}{2}=1\frac{1}{2}$$

Test Yourself 6

Divide out common factor wherever possible and express your answers in simplest form.

1. $\frac{4}{5} \times \frac{3}{6} =$
2. $\frac{2}{4} \times \frac{8}{12} \times \frac{7}{1} =$
3. $\frac{3}{4} \div \frac{3}{8} =$
4. $\frac{5}{2} \div \frac{3}{6} =$
5. $\frac{8}{9} \times \frac{3}{4} \times \frac{1}{2} =$
6. $\frac{7}{8} \div \frac{2}{3} =$
7. $\frac{4}{16} \times \frac{8}{12} \times \frac{10}{13} =$
8. $\frac{1}{6} \times \frac{7}{6} \times \frac{12}{3} =$
9. $\frac{3}{7} \div \frac{9}{4} =$
10. $\frac{2}{3} \div \frac{2}{3} =$

The fraction bar in a fraction means "divided by." To rename a fraction as a decimal, follow through on the division.

$$\frac{4}{5} = 4 \div 5 = 0.8$$

To rename a decimal as a percent, multiply by 100, move the decimal point two places to the right, and attach a percent sign.

$$0.8 = 80\%$$

Test Yourself 7

Rename each fraction, first as a decimal to three places, and then as a percent.

1. $\frac{2}{4}$
2. $\frac{7}{8}$
3. $\frac{5}{6}$
4. $\frac{6}{8}$
5. $\frac{3}{4}$
6. $\frac{2}{3}$
7. $\frac{3}{5}$
8. $\frac{4}{10}$
9. $\frac{1}{4}$
10. $\frac{2}{5}$

PERCENTAGES

One percent is one hundredth of something. The last syllable of the word *percent, -cent*, is the name we give to one hundredth of a dollar.

One percent of $1.00, then, is one cent. Using decimal notation, we can write one cent as $0.01, five cents as $0.05, twenty-five cents as $0.25, and so forth.

Twenty-five cents represents twenty-five hundredths of a dollar. Rather than say that something is so many hundredths of something else, we use the word percent. Twenty-five cents, then, is twenty-five percent of a dollar. We use the symbol % to stand for percent.

Percentage ("hundredths of") is a convenient and widely used way of measuring all sorts of things. By measuring in hundredths, we can be very precise and notice very small changes.

Percentage is not limited to comparing other numbers to 100. You can divide any number into hundredths and talk about percentage.

Example: Find 1% of 200.

Solution: 1% of 200 is one hundredth of 200.

$200 \div 100 = 2$

Using decimal notation, we can calculate one percent of 200 by:

$$200 \times 0.01 = 2$$

Similarly, we can find a percentage of any number we choose by multiplying it by the correct decimal notation. For example:

Five percent of 50: $0.05 \times 50 = 2.5$
Three percent of 150: $0.03 \times 150 = 4.5$
Ten percent of 60: $0.10 \times 60 = 6.0$

All percentage measurements are not between one percent and 100 percent. We may wish to consider less than one percent of something, especially if it is very large.

For example, if you were handed a book 1,000 pages long and were told to read one percent of it in 5 minutes, how much would you have to read?

$$1000 \times 0.01 = 10 \text{ pages}$$

Quite an assignment! You might bargain to read one half of one percent, or one-tenth of one percent in the 5 minutes allotted to you.

Using decimal notation, we write one-tenth of one percent as 0.001, the decimal number for one thousandth. If you remember that a percent is one hundredth of something, you can see that one tenth of that percent is equivalent to one thousandth of the whole.

NOTE

There is a relationship between decimals, fractions, and percents. The following notes will help you to convert numbers from one of these forms to another:

1. To change a % to a decimal, remove the % sign and divide by 100.
2. To change a decimal to a %, add the % sign and multiply by 100.
3. To change a % to a fraction, remove the % sign and divide by 100.
4. To change a fraction to %, multiply by 100 and add the % sign.

In percent notation, one tenth of one percent is written as 0.1%. On high school entrance exams, students often mistakenly think that 0.1% is equal to 0.1. As you know, 0.1% is really equal to 0.001.

Sometimes we are concerned with more than 100% of something. But, you may ask, since 100% constitutes all of something, how can we speak of *more* than all of it?

Where things are growing, or increasing in size or amount, we may want to compare their new size to the size they once were. For example, suppose we measured the heights of three plants to be 6 inches, 9 inches, and 12 inches one week and discover a week later that the first plant is still 6 inches tall but the second and third ones are now 18 inches tall.

The 6-inch plant grew *zero percent* because it didn't grow at all.
The second plant *added 100%* to its size. It doubled in height.
The third plant *added 50%* to its height.

We can also say:

The first plant is 100% of its original height.
The second plant grew to 200% of its original height.
The third plant grew to 150% of its original height.

Here are some common percentage and fractional equivalents you should remember:

- Ten percent (10%) is one tenth $\left(\frac{1}{10}\right)$, or 0.10.
- Twelve and one-half percent (12.5%) is one eighth $\left(\frac{1}{8}\right)$, or 0.125.
- Twenty percent (20%) is one fifth $\left(\frac{1}{5}\right)$, or 0.20.
- Twenty-five percent (25%) is one quarter $\left(\frac{1}{4}\right)$, or 0.25.
- Thirty-three and one-third percent ($33\frac{1}{3}\%$) is one third $\left(\frac{1}{3}\right)$, or 0.333.
- Fifty percent (50%) is one half $\left(\frac{1}{2}\right)$, or 0.50.
- Sixty-six and two-thirds percent ($66\frac{2}{3}\%$) is two thirds $\left(\frac{2}{3}\right)$, or 0.666.
- Seventy-five percent (75%) is three quarters $\left(\frac{3}{4}\right)$, or 0.75.

Caution: When solving problems involving percentages, be careful of common errors:

- **Read the notation carefully.** 0.50% is *not* fifty percent, but one half of one percent.
- When solving problems for percentage increases or decreases in size, **read the problems carefully.**

- **Use common sense.** If you wish to find less than 100% of a number, your result will be smaller than the number you started with. For example, 43% of 50 is less than 50. Using common sense works in the other direction as well. For example, 70 is 40% of what number? The number you are looking for must be larger than 70, since 70 is only $\frac{40}{100}$ of it. Moreover, you can estimate that the number you are looking for will be a little more than twice as large as 70, since 70 is almost half (50%) of that number.

To find a percent of a number, rename the percent as a decimal and multiply the number by it.

Example: What is 5% of 80?

Solution: 5% of 80 $= 80 \times 0.05 = 4$

To find out what a number is when a percent of it is given, rename the percent as a decimal and divide the given number by it.

Example: 5 is 10% of what number?

Solution: $5 \div 0.10 = 50$

To find what percent one number is of another number, create a fraction by placing the part over the whole. Simplify the fraction if possible, then rename it as a decimal (remember the fraction bar means *divided by,* so divide the numerator by the denominator) and rename the answer as a percent by multiplying by 100, moving the decimal point two places to the right.

Example: 4 is what percent of 80?

Solution: $\frac{4}{80} = \frac{1}{20} = 0.05 = 5\%$

Test Yourself 8

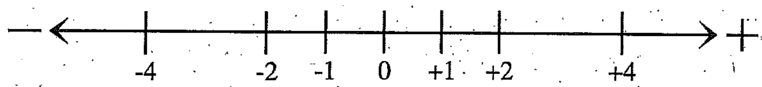
1. 10% of 32 =
2. 8 is 25% of what number?
3. 12 is what percent of 24?
4. 20% of 360 =
5. 5 is what percent of 60?
6. 12 is 8% of what number?
7. 6% of 36 =
8. 25 is 5% of what number?
9. 70 is what percent of 140?
10. What percent of 100 is 19?

ALGEBRA

If you are finishing the eighth grade this year, you might not yet have had a formal algebra class. Nevertheless, you have probably used algebraic terms and expressions, and you have probably solved simple equations. This section will review the skills you have acquired so far and will show you the kinds of questions you can expect to find on a high school entrance examination.

Signed Numbers

The number line exists to both sides of zero. Each positive number on the right of zero has a negative counterpart to the left of zero. The number line below shows the location of some pairs of numbers (+4, −4; +2, −2; +1, −1).



TIP

To add signed numbers with the same sign, add the magnitudes of the numbers and keep the same sign. To add signed numbers with different signs, subtract the magnitudes of the numbers and use the sign of the number with the greater magnitude.

Because each number of a pair is located the same distance from zero (though in different directions), each has the same absolute value. Two vertical bars symbolize absolute value:

$$|+4| = |-4| = 4$$

The absolute value of +4 equals the absolute value of −4. Both are equivalent to 4. If you think of absolute value as the distance from zero, regardless of direction, you will understand it easily. The absolute value of any number, positive or negative, is always expressed as a positive number.

Addition of Signed Numbers

When two oppositely signed numbers having the same absolute value are added, the sum is zero.

$$(+10) + (-10) = 0$$

$$(-1.5) + (+1.5) = 0$$

$$(-0.010) + (+0.010) = 0$$

$$(+\frac{3}{4}) + (-\frac{3}{4}) = 0$$

If one of the two oppositely signed numbers is greater in absolute value, the sum is equal to the amount of that excess and carries the same sign as the number having the greater absolute value.

$$(+2) + (-1) = +1 \qquad (-2.5) + (+2.0) = -0.5$$

$$(+8) + (-9) = -1 \qquad (-\frac{3}{4}) + (+\frac{1}{2}) = -\frac{1}{4}$$

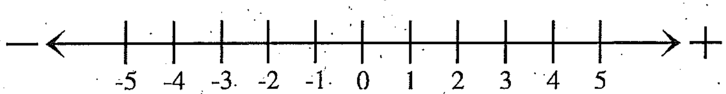
Test Yourself 9

1. $(+5) + (+8) =$
2. $(+6) + (-3) =$
3. $(+4) + (-12) =$
4. $(-7) + (+2) =$
5. $(-21) + (-17) =$
6. $(-9) + (-36) =$
7. $(+31) + (-14) =$
8. $(-16.3) + (-12.5) =$
9. $(-8\frac{1}{2}) + (+4\frac{1}{4}) =$
10. $(+66) + (-66) =$

Subtraction of Signed Numbers

Subtraction is the operation that finds the difference between two numbers, including the difference between signed numbers.

When subtracting signed numbers, it is helpful to refer to the number line.



For example, if we wish to subtract +2 from +5, we can use the number line to see that the difference is +3. We give the sign to the difference that represents the direction we are moving along the number line from the number being subtracted to the number from which you are subtracting. In this case, because we are subtracting +2 from +5, we count three units in a positive direction from +2 to +5 on the number line.

When subtracting signed numbers:

- The distance between the two numbers gives you the absolute value of the difference.
- The direction you have to move from the number being subtracted to get to the number from which you are subtracting gives you the sign of the difference.

Example: Subtract −3 from +5.

Solution: Distance on the number line between −3 and +5 is 8 units.
Direction is from negative to positive—a positive direction.
Answer is +8.

Example: Subtract −6 from −8.

Solution: Distance on number line between −6 and −8 is 2 units. Direction is from −6 to −8—a negative direction. Answer is −2.

Example: Subtract +1.30 from −2.70.

Solution: Distance between them on the number line is 4.0. Direction is from +1.30 to −2.70—a negative direction. Answer is −4.0.

NOTE

Change the sign of the number being subtracted and follow the rules for addition.

A quick way to subtract signed numbers accurately involves placing the numbers in columns, reversing the sign of the number being subtracted and then adding the two.

Example: Subtract +26 from +15.

Solution:

$$\begin{array}{rcr} +15 & = & +15 \\ \underline{-+26} & = & \underline{-26} \\ & = & -11 \end{array}$$

Example: Subtract −35 from +10.

Solution:

$$\begin{array}{rcr} +10 & = & +10 \\ \underline{--35} & = & \underline{+35} \\ & = & +45 \end{array}$$

Notice that in each of the examples, the correct answer was found by reversing the sign of the number being subtracted and then adding.

Test Yourself 10

1. $(-6) - (-12) =$
2. $(+17) - (-8) =$
3. $(+45) - (+62) =$
4. $(-34) - (+21) =$
5. $(+4) - (-58) =$
6. $(+75) - (+27) =$
7. $(-12.6) - (-5.3) =$
8. $\left(-15\frac{1}{4}\right) - \left(+26\frac{1}{2}\right) =$
9. $(-35) - (+35) =$
10. $(+56.1) - (+56.7) =$

Multiplication of Signed Numbers

Signed numbers are multiplied as any other numbers would be, with the following exceptions:

The product of two negative numbers is positive.

$$(-3) \times (-6) = +18$$

The product of two positive numbers is positive.

$$(+3.05) \times (+6) = +18.30$$

The product of a negative and positive number is negative.

$$\left(+4\frac{1}{2}\right) \times (-3) = -13\frac{1}{2}$$
$$(+1) \times (-1) \times (+1) = -1$$

Test Yourself 11

1. $(+5) \times (+8) =$
2. $(+12) \times (-3) =$
3. $(-6) \times (-21) =$
4. $(-4) \times (-10) =$
5. $(+3.3) \times (-5.8) =$
6. $(-7.5) \times (+4.2) =$
7. $\left(-6\frac{1}{2}\right) \times \left(-7\frac{1}{4}\right) =$
8. $(+9) \times (-1) =$
9. $(0) \times (-5.7) =$
10. $(-12) \times (-12) =$

Division of Signed Numbers

As with multiplication, the division of signed numbers requires you to observe three simple rules:

1. When dividing a positive number by a negative number, the result is negative.

$$(+6) \div (-3) = -2$$

2. When dividing a negative number by a positive number, the result is negative.

$$(-6) \div (+3) = -2$$

3. When dividing a negative number by a negative number or a positive number by a positive number, the result is positive.

$$(-6) \div (-3) = +2$$
$$(+6) \div (+3) = +2$$

NOTE

If the signs are the same, the quotient is positive. If the signs are different, the quotient is negative.

Test Yourself 12

1. $(+3) \div (-1) =$
2. $(+36) \div (+12) =$
3. $(-45) \div (-9) =$
4. $(-75) \div (+3) =$
5. $(+5.6) \div (-0.7) =$
6. $(-3.5) \div (-5) =$
7. $\left(+6\frac{1}{2}\right) \div \left(+3\frac{1}{4}\right) =$
8. $(-8.2) \div (-1) =$
9. $\left(+12\frac{1}{2}\right) \div \left(-12\frac{1}{2}\right) =$
10. $(0) \div (-19.6) =$

EQUATIONS

An equation is an equality. The values on either side of the equal sign in an equation must be equal. In order to learn the value of an unknown in an equation, do the same thing to both sides of the equation so as to leave the unknown on one side of the equal sign and its value on the other side.

Example: $x - 2 = 8$

Solution: Add 2 to both sides of the equation:

$$x - 2 + 2 = 8 + 2$$
$$x = 10$$

Example: $5x = 25$

Solution: Divide both sides of the equation by 5:

$$\frac{{}^{1}\cancel{5}x}{\cancel{5}_{1}} = \frac{25}{5}$$
$$x = 5$$

Example: $y + 9 = 15$

Solution: Subtract 9 from both sides of the equation:

$$y + 9 - 9 = 15 - 9$$
$$y = 6$$

Example: $a \div 4 = 48$

Solution: Multiply both sides of the equation by 4:

$$^{1}\cancel{4}\left(\frac{a}{\cancel{4}_{1}}\right) = 48 \times 4$$

$$a = 192$$

Sometimes more than one step is required to solve an equation.

Example: $6a \div 4 = 48$

Solution: First, multiply both sides of the equation by 4:

$$\frac{6a}{4} \times \frac{4}{1} = 48 \times 4$$

$$6a = 192$$

Then divide both sides of the equation by 6:

$$\frac{^{1}\cancel{6}a}{\cancel{6}_{1}} = \frac{192}{6}$$

$$a = 32$$

Test Yourself 13

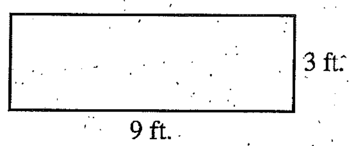
Solve for x.

1. $x + 13 = 25$
2. $4x = 84$
3. $x - 5 = 28$
4. $x \div 9 = 4$
5. $3x + 2 = 14$
6. $\frac{x}{4} - 2 = 4$
7. $10x - 27 = 73$
8. $2x \div 4 = 13$
9. $8x + 9 = 81$
10. $2x \div 11 = 6$

GEOMETRY

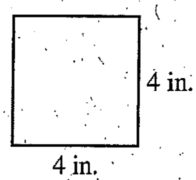
Area of Plane Figures

Area is the space enclosed by a plane (flat) figure. A rectangle is a plane figure with four right angles. Opposite sides of a rectangle are of equal length and are parallel to each other. To find the area of a rectangle, multiply the length of the base of the rectangle by the length of its height. Area is always expressed in square units.



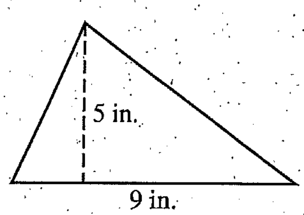
$$A = bh$$
$$A = 9 \text{ ft.} \times 3 \text{ ft.}$$
$$A = 27 \text{ sq. ft.}$$

A square is a rectangle in which all four sides are the same length. The area of a square is found by squaring the length of one side, which is exactly the same as multiplying the square's length by its width.



$$A = s^2$$
$$A = 4 \text{ in.} \times 4 \text{ in.}$$
$$A = 16 \text{ sq. in.}$$

A triangle is a three-sided plane figure. The area of a triangle is found by multiplying the base by the altitude (height) and dividing by two.

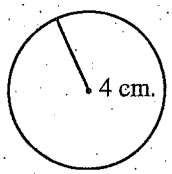


$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(9 \text{ in.})\ (5 \text{ in.}) = \frac{45}{2}$$
$$A = 22\frac{1}{2} \text{ sq. in.}$$

A circle is a perfectly round plane figure. The distance from the center of a circle to its rim is its radius. The distance from one edge to the other through the center is its diameter. The diameter is twice the length of the radius.

Pi (π) is a mathematical value equal to approximately 3.14, or $\frac{22}{7}$. Pi (π) is frequently used in calculations involving circles. The area of a circle is found by squaring the radius and multiplying it by π. You may leave the area in terms of pi unless you are told what value to assign π.

Don't confuse the two formulas for calculating the circumference and the area of circles. A good way to keep them straight is to remember the square in πr^2. It should remind you that area must be in square units.

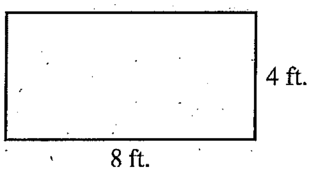


$$A = \pi r^2$$
$$A = \pi \text{ (4 cm.)}^2$$
$$A = 16\pi \text{ sq. cm.}$$

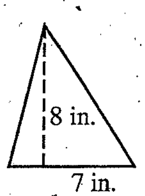
Test Yourself 14

Find the area of each figure.

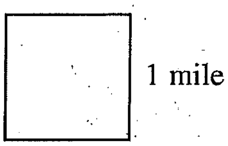
1.



2.



3.



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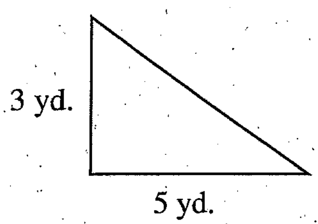
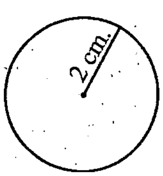
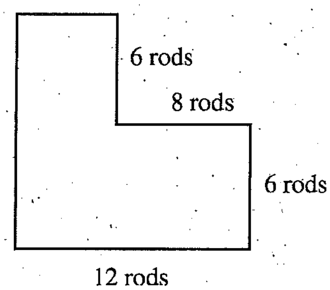


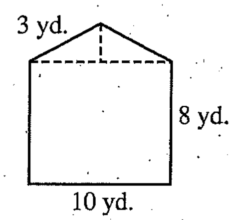
5.



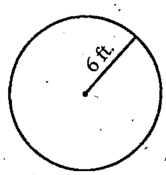
6.



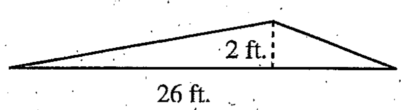
7.



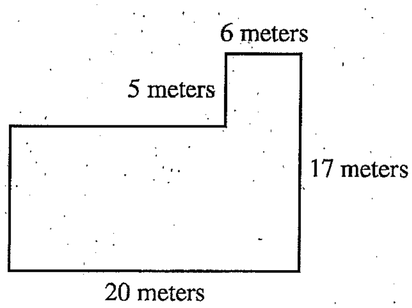
8.



9.

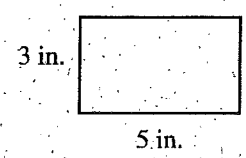


10.

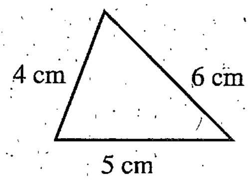


Perimeter of Plane Figures

The *perimeter* of a plane figure is the distance around the outside. To find the perimeter of a polygon (a plane figure bounded by straight lines), just add the lengths of the sides.

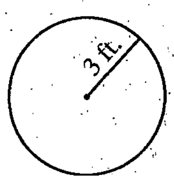


$P = 3 \text{ in.} + 5 \text{ in.} + 3 \text{ in.} + 5 \text{ in.}$
$= 16 \text{ in.}$



$P = 4 \text{ cm} + 6 \text{ cm} + 5 \text{ cm}$
$= 15 \text{ cm}$

The perimeter of a circle is called the circumference. The formula for the circumference of a circle is πd or $2\pi r$, which are both, of course, the same thing.

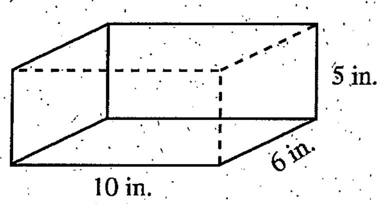


$C = 2 \times 3 \times \pi = 6\pi$

Volume of Solid Figures

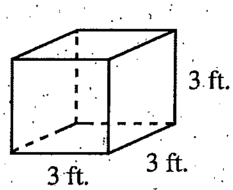
The volume of a solid figure is the measure of the space within. To figure the volume of a solid figure, multiply the area of the base by the height or depth.

The volume of a rectangular solid is length × width × height. Volume is always expressed in cubic units.



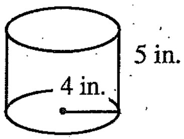
$V = lwh$
$V = (10 \text{ in.})(6 \text{ in.})(5 \text{ in.})$
$V = 300 \text{ cu. in.}$

The volume of a cube is the cube of one side.



$V = s^3$
$V = (3 \text{ ft.})^3$
$V = 27 \text{ cu. ft.}$

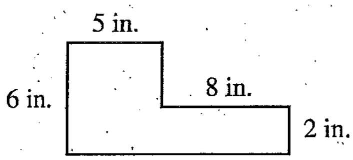
The volume of a cylinder is the area of the circular base (πr^2) times the height.



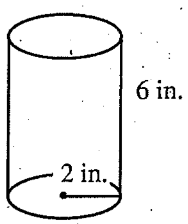
$$V = \pi r^2 h$$
$$V = \pi\ (4\text{ in.})^2\ (5\text{ in.})$$
$$V = \pi(16)(5) = 80\pi\text{ cu. in.}$$

Test Yourself 15

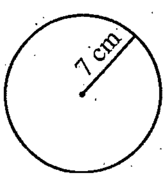
1. Find the perimeter.



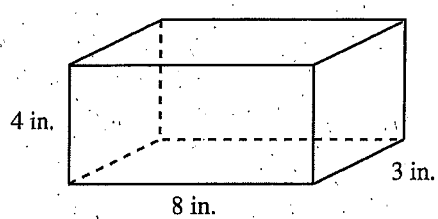
2. Find the volume.



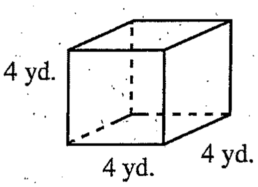
3. Find the circumference.



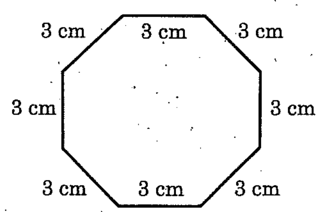
4. Find the volume.



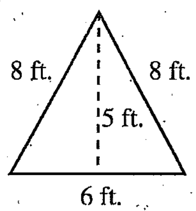
5. Find the volume.



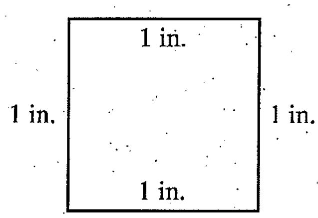
6. Find the perimeter.



7. Find the perimeter.

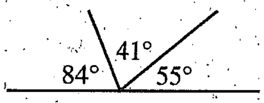


8. Find the perimeter.

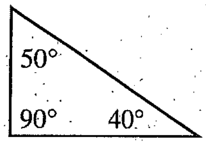


Angles

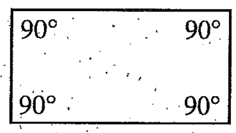
The sum of the angles of a straight line is 180°.



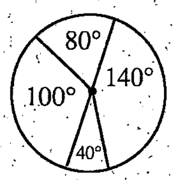
The sum of the angles of a triangle is 180°.



The sum of the angles of a rectangle is 360°.

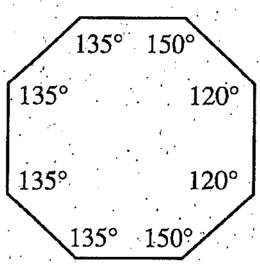


The sum of the angles of a circle is 360°.



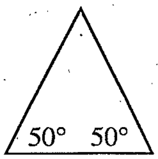
The sum of the angles of a polygon of n sides is $(n - 2)180°$.

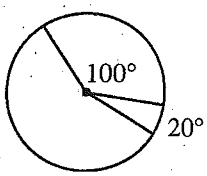
$$(8 - 2)(180°) = 6 \times 180° = 1080°$$

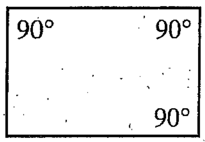


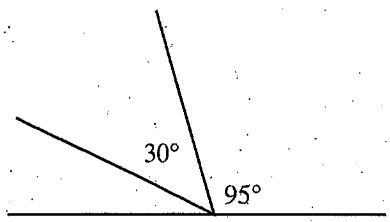
Test Yourself 16

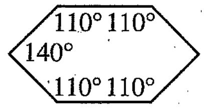
What is the size of the unlabeled angle?

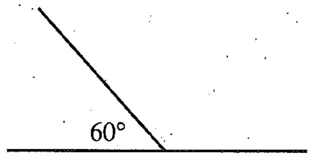
1. 

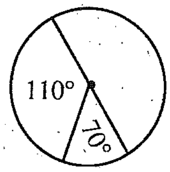
2. 

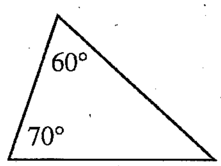
3. 

4. 

5. 

6. 

7. 

8. 

COORDINATE GEOMETRY

Coordinate geometry is used to locate and graph points and lines on a plane.

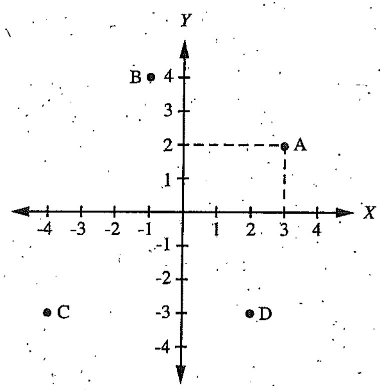
The coordinate system is made up of two perpendicular number lines that intersect at 0. Any point on the plane has two numbers, or coordinates, that indicate its location relative to the number lines.

The x-coordinate (abscissa) is found by drawing a vertical line from the point to the horizontal number line (the x-axis). The number found on the x-axis is the abscissa.

The y-coordinate (ordinate) is found by drawing a horizontal line from the point to the vertical number line (the y-axis). The number found on the y-axis is the ordinate.

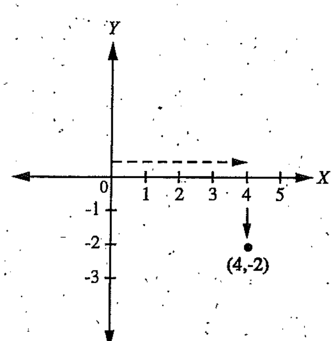
The two coordinates are always written in the order (x,y).

The x-coordinate of point A is 3. The y-coordinate of point A is 2. The coordinates of point A are given by the ordered pair (3,2). Point B has coordinates $(-1,4)$. Point C has coordinates $(-4,-3)$. Point D has coordinates $(2,-3)$.



To graph a point whose coordinates are given, first locate the x-coordinate on the x-axis, then from that position move vertically the number of spaces indicated by the y-coordinate.

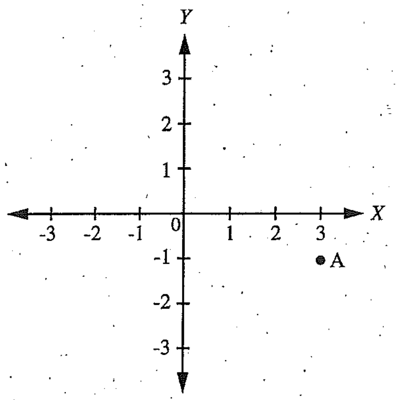
To graph $(4,-2)$, locate 4 on the x-axis, then move -2 spaces vertically (2 spaces down) to find the given point.



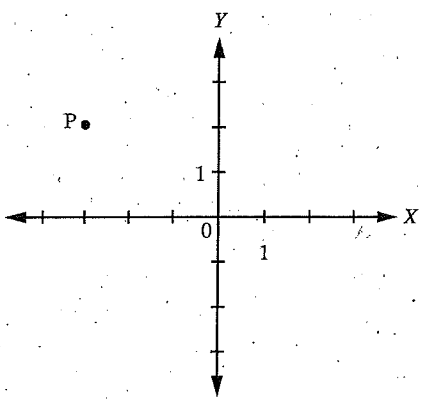
The point at which the x-axis and the y-axis meet has coordinates (0,0) and is called the origin. Any point on the y-axis has 0 as its x-coordinate. Any point on the x-axis has 0 as its y-coordinate.

Test Yourself 17

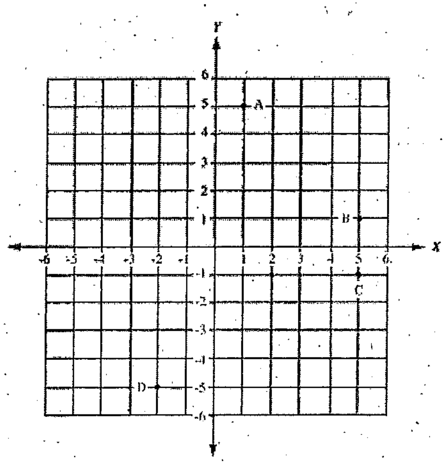
1. In the graph below, the coordinates of point A are



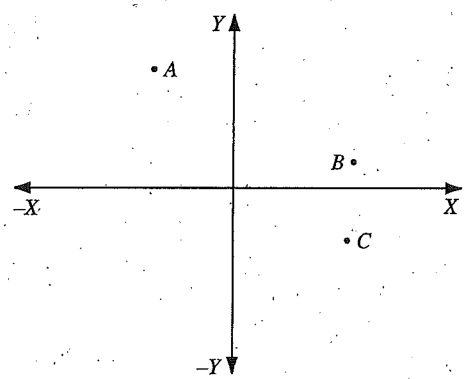
2. The coordinates of point P on the graph are



3. Which point is named by the ordered pair (5, 1)?



4. Which point might possibly have the coordinates (2,−3)?



5. The point with the coordinates (3,0) is